



- [Philica front page](#) [Search](#)
- [About Philica](#) [Take the](#)
- [tour](#)
- [Publish your work](#) [Work](#)
- [needing review](#) [Most](#)
- [popular entries](#) [Highest-](#)
- [rated entries](#) [Recent](#)
- [reviews](#)
- [How to cite Philica](#) [FAQs](#)
- [Support Philica](#) [Contact us](#)

A QED-model for the Energy of the Vacuum and an Explanation of its Conversion into Mechanical Energy

[Claus W. Turtur](#)   (Fachbereich Elektrotechnik, University of Applied Sciences Braunschweig-Wolfenbuettel)

Published in physic.philica.com

Abstract

The energy density of the vacuum is still one of the unsolved questions in physics. Principally it should be possible to calculate this energy density by summing up the energy of all zero point oscillations of the vacuum. The problem is that this sum leads to a divergent improper integral. An approach to a solution is found within the theory of Geometrodynamics, however this approach is regarded sceptical nowadays and it is in contradiction with measurements of Astrophysics. The present work introduces a new solution for the convergence problem of the improper integral on the basis of Quantum electrodynamics (where improper integrals are solved), coming to values appearing realistic. The only necessary postulate is: It is known that the speed of propagation of electromagnetic waves is influenced by electric and magnetic DC-fields. The postulate is now to assume, that the zero point oscillations of the vacuum display the same behaviour. But the article is not restricted to theoretical calculations of the energy of the zero point oscillations of the vacuum; it also goes back to the experimental verification and to the utilization of those zero point oscillations in laboratory giving a theoretical (quantumelectrodynamical) explanation of the operation of the experiment, which already has been conducted successfully.

Article body

1. Introduction

As generally known in Quantum theory, the eigen-values of the energy of electromagnetic waves are given as $(n+1/2) \cdot (h/2\pi) \cdot \omega$, where "n" is number of photons, calculated as the eigen-values of $\hat{a}^\dagger \hat{a}$ with regard to the wave function ψ_n in the equation $\hat{a}^\dagger \hat{a} \psi_n = n \cdot \psi_n$ (with \hat{a}^\dagger being the operator of particle creation and \hat{a} being the operator of particle annihilation). As long as no particles are present, we have $n=0$, and the eigen-values of the energy of the ideal (physical) vacuum $|0\rangle$ (in Dirac's notation) are found by integration over all frequencies ω respectively over all wave vectors \mathbf{k} in the \mathbf{k} -space (letters in bold symbolize vectors) leading to

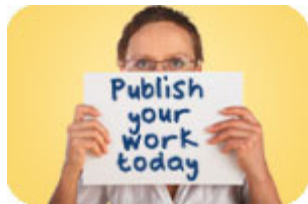
$$(01) \quad E = \int \frac{1}{2} \hbar \omega d^3 \vec{k}$$

(without consideration of polarization) [1].

We know that this integral is divergent, because for small wavelength $\lambda \rightarrow 0$ (which fit well into every small volume), the absolute values of the wave vector $|\mathbf{k}| = (k_x^2 + k_y^2 + k_z^2)^{1/2}$ as well as the frequency ω go to infinity. This brings the problem that the integral in (1) leads to an infinite energy density. Normally this energy density is treated as a constant without significance to physics, and which is eliminated by setting the zero point of energy to the ground state $|0\rangle$ of the vacuum [2]. The production of one photon $\hat{a}^\dagger |0\rangle = |1\rangle$ results in the excited state of a harmonic oscillator, of which the eigen-value of the energy is by $1 \cdot (h/2\pi) \cdot \omega = (h/2\pi) \cdot c |\mathbf{k}|$ above the energy of the ground state [3].

The propagation of a photon is known to follow the speed of light. Because the propagation of electrostatic and magnetic fields are understood as the exchange of photons, the logical consequence should be, that electric and magnetic DC-fields should also follow this speed of propagation. Assigning this conception to DC-fields is not usual for everybody, but we will find further justification in the following chapters with arguments within the theory of Relativity and with arguments within Quantum theory.

The fact, that even the ground state $|0\rangle$ of the empty vacuum contains the energy of harmonic oscillations of electromagnetic waves, is the reason that they got the name "zero point oscillations". Their energy according to (1) defines



We aim to suit all browsers, but recommend Firefox particularly:

the vacuum-energy of the ground state. If we want to have access to this energy (and to convert it into a classical form of energy), we have to understand their nature. A well known example for a force coming out of the vacuum-energy is the Casimir-force [4,5,6,7], which is explained on the basis of zero point oscillations. This explanation is based on the analysis of the influence of two ideally conducting (metallic) plates onto the spectrum of zero point oscillations of the vacuum. The free vacuum (i.e. without those plates) consists of a continuous spectrum of all imaginable wavelengths, whereas the space between the plates only contains a discrete spectrum of resonant (standing) waves, because the plates act as reflectors with a field strength of zero at the surface of each plate, defining nodal points of the oscillation. From the energy-difference between those both spectra, Casimir deduces the energy density and the force between the plates.

This arises the expectation that the conversion of vacuum-energy into mechanical energy as reported in [8,9] can be understood in analogy to the Casimir-effect. Its invention came out of this analogy. If this energy conversion shall be done in a perpetual process, we have to find a possibility to move the plates relatively to each other without alteration of the distance between them. So there is some similarity with the Casimir-effect, but there is also an important difference. The solution can be found in a construction of rotating plates, as they are presented in chapter 2.

Although the Casimir-effect helped the author to invent a machine which verifies the existence of the energy of the zero point oscillations and converts it into mechanical energy, it was clear from the very beginning of the development, that the metal plates and the vacuum, necessary for the Casimir-effect are not enough for the endless conversion of vacuum-energy. A further item, like an electric or a magnetic field would be necessary in addition to the components of the Casimir-effect, and these further items have to interact with the zero point oscillations.

The experiment which finally succeeded in converting vacuum-energy into classical mechanical energy [8,9] confirms this approach. The first explanation of the functioning principle of the energy converting rotor was given on the basis of an electrostatic field within classical electrodynamics [10, 11]. The logical connection between the zero point oscillations and classical electrodynamics is topic of this article here. A central aspect thereby is the mechanism how the electric and the magnetic fields propagate in the vacuum (i.e. into the space). The crucial question is: How do those fields influence the zero point oscillations. This question will also be answered in the following chapters.

2. Hitherto existing model of the conversion of vacuum-energy

Before we answer the crucial concluding question of chapter 1, we want to recapitulate the basics of the model according to [11] (within classical physics), which showed the way how to convert vacuum-energy into classical energy:

Electric and magnetic fields as regarded in classical electrodynamics are treated to be "everywhere in space at the same moment" [12]. This means, that normally their time dependent propagation into the space is not taken into consideration, but only their presence. For most of the technical and practical applications of electrodynamics (with typical distances inside the laboratory and velocities negligible in comparison with the speed of light) this is fully sufficient. But as a matter of principle this is in clear contradiction with the theory of Relativity [13], according to which the propagation of the field strength has to respect at least the limit of the speed of light [14]. However there is also one issue of electrodynamics taking the finite speed of propagation into account, this is the retarded Lienard-Wiechert-potential [15]. In this way we see, that even classical electrodynamics does not ignore completely the finite speed of propagation of the DC-field.

Thus it appears sensible, to assume the speed of propagation of the electric and the magnetic field to be identically with the speed electromagnetic waves. But this conception has extensive consequences, as for instance the conclusion, that electric and magnetic fields dispense energy to the vacuum during their propagation, as can be seen from the following demonstration:

Let us regard an electric field, which is emitted by a point charge "Q" (see fig.1), and let us trace this field during its propagation into the space. We begin our considerations with a sphere around Q, having the radius x_1 at the time $t=0$. At a moment $t>0$ (later in time), the field within this sphere will have been propagated with the speed "c" into a sphere with the radius $x_1+c\cdot\Delta t$. This means, that during the time interval Δt , the charge has emitted the same amount of energy, as contained inside the spherical shell from x_1 to $x_1+c\cdot\Delta t$, because this is the amount of energy by which the total energy of the field was enhanced during the time interval Δt . Because this amount of energy is definitely larger than zero, we can conclude, that the charge Q really has emitted energy. This is one of the consequences coming from the finite speed of propagation of the DC-field.

Furthermore we consider the propagation of this spherical shell from x_1 to $x_1+c\cdot\Delta t$ into the space until its radii reach from x_2 to $x_2+c\cdot\Delta t$. Let us nominate the time t_2 at which the shell reaches this volume (with t_2 being later than Δt). By comparing the amount of energy within the both shells from x_1 to $x_1+c\cdot\Delta t$ and from x_2 to $x_2+c\cdot\Delta t$, we will discover the surprising fact, that the spherical shell has lost some of its field energy during propagation.

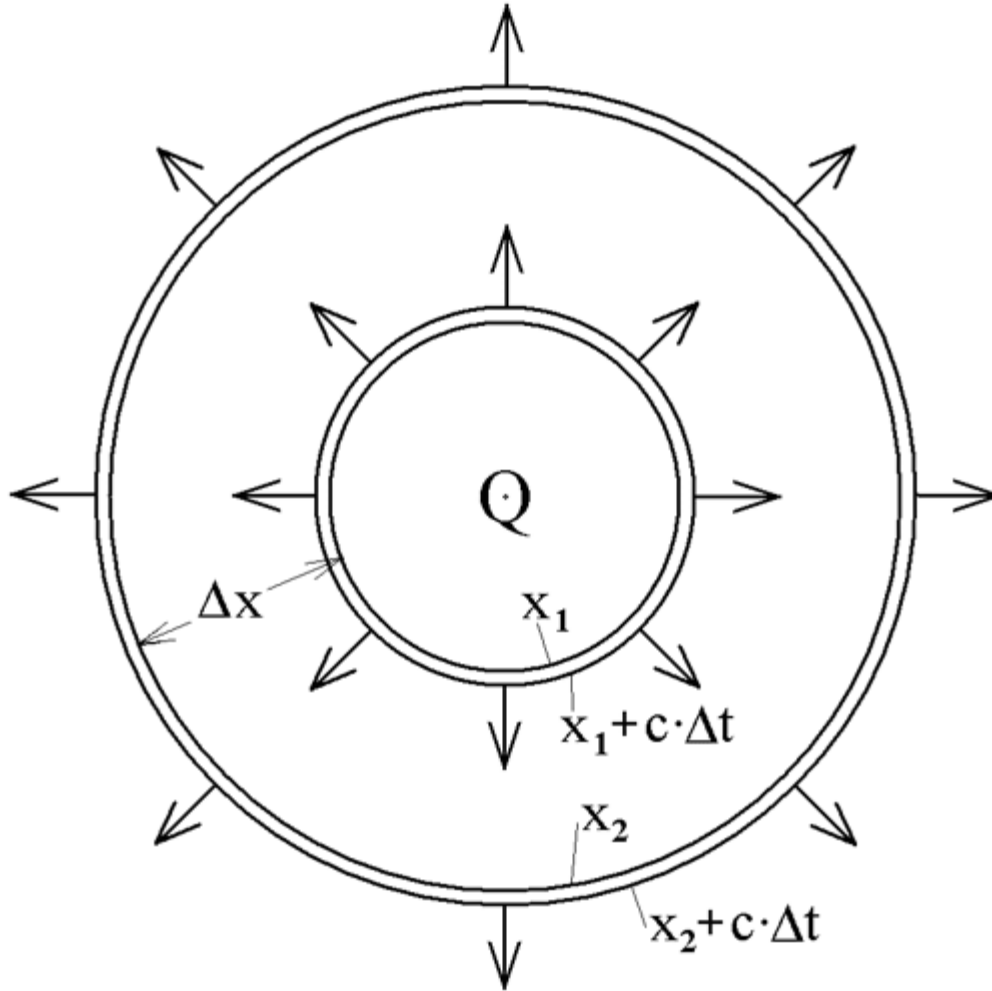


Fig. 1: Illustration of a spherical shell, which contains a certain amount of field energy of the electrostatic field of a point charge "Q". Sense of this sketch is to trace field energy during its propagation into the space. Additional remark: Sometimes there was a counter-argument against the conception of finite speed of propagation of the field, which said, that all point charges exist already since the "big bang" at the beginning of the world. Actually this argument does not affect our considerations, but it only expresses, that the sphere with "Q" in its centre can very large, so that it can have the radius equal to the speed of light multiplied with the age of the universe.

The calculation which proves these statements about the field energy during the propagation of the field shall be remembered briefly. The energy density of the field of a point charge with central symmetry is

$$u = \frac{\epsilon_0}{2} \cdot |\vec{E}|^2 = \frac{Q^2}{32\pi^2 \epsilon_0 r^4}$$

Consequently the energy within the inner shell from x_1 to $x_1 + c \cdot \Delta t$ is

$$(02) \quad E_{\text{inner shell}} = \int_{\text{spherical shell}} u(\vec{r}) dV = \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi} \int_{r=x_1}^{x_1+c \cdot \Delta t} \frac{Q^2}{32\pi^2 \epsilon_0 r^4} \cdot r^2 \cdot \sin(\vartheta) dr d\vartheta d\varphi = \frac{Q^2}{8\pi \epsilon_0} \cdot \frac{c \cdot \Delta t}{(x_1 + c \cdot \Delta t) \cdot x_1}$$

and the energy within the outer shell from x_2 to $x_2+c\cdot\Delta t$ is

$$(03) \quad E_{\text{outer shell}} = \int_{\text{spherical shell}} u(\vec{r}) dV = \int_{\varphi=0}^{2\pi} \int_{\vartheta=0}^{\pi} \int_{r=x_2}^{x_2+c\cdot\Delta t} \frac{Q^2}{32\pi^2 \epsilon_0 r^4} \cdot r^2 \cdot \sin(\vartheta) dr d\vartheta d\varphi = \frac{Q^2}{8\pi\epsilon_0} \cdot \frac{c \cdot \Delta t}{(x_1 + \Delta x + c \cdot \Delta t) \cdot (x_1 + \Delta x)}$$

Obviously the energy contents within the spherical shell decreased when the field propagated $x_1 \dots x_1+c\cdot\Delta t$ to $x_2 \dots x_2+c\cdot\Delta t$. The propagation can take place in vacuum, so there is only the vacuum (i.e. the space), which can absorb the energy. (If there are additionally some molecules within the space, we want to neglect their influence on the process).

On the other hand, the field source, which is the charge Q , permanently feeds the field with energy without being altered or exhausted at all. But from where does the field source get this energy. It must be supplied from the vacuum, because the field source is in contact with nothing else but only the vacuum. Finally we notice that there must be a circulation of energy. The field source gets energy from the space (the vacuum) and emits it into the vacuum. But the vacuum takes part of this back by absorbing it from the field.

This surprising circulation of energy can be made plausible on the basis of the structure of the vacuum as it is understood in Quantum mechanics, Quantum electrodynamics and Quantum field theory. The conception is presented in the further course of the article. By the way, the conception of the vacuum absorbing field energy is not only applicable for the electric field but also for the magnetic field [16]. Both of them undergo a similar energy circulation process. Thus our Quantum theoretical model is being developed in a way that it explains the propagation of both types of fields (otherwise it would not be applicable to electromagnetic waves). And the mechanism has to leads us to the same energy density of the vacuum for both types of fields, because there is only one vacuum responsible for the propagation of both fields.

For the sake of better comprehensibility, we now want to have a short glance to the experiment, with which the existence and the conversion of the vacuum energy (within the zero point oscillations) was already successfully performed [8]. The central point is to extract energy from one of the energy circulations described above. A possible setup therefore is shown in fig.2, where a rotor made from ideal conducting plates is mounted rotatable (supplied by an axis) below an ideal conducting slab, which acts as a field source. The field source is brought to a certain electric potential just by putting some electrical charge onto its surface. Consequently it permanently emits field energy, which it gets from the vacuum. The rotor is connected electrically to ground and thus it permanently extracts energy from the energy flux emanated by the field source. This makes the rotor spin. In chapter 3 and 4 we will see how this can happen.

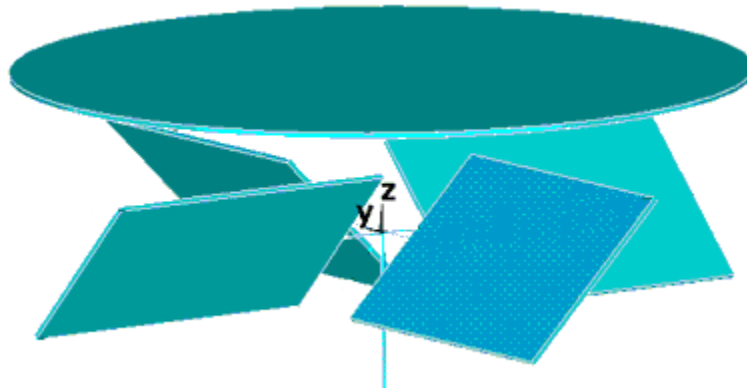


Fig. 2: Principle sketch of the experimental setup for the conversion of vacuum energy into mechanical-energy.

Although the existence of the described energy circulation is concluded logically from basic statements of classical electrodynamics, its background will be evident only from the inner structure of the vacuum. From the basis of this understanding, the really observed conversion of vacuum energy into mechanical energy is explained.

In order to prepare the microscopic model of energy conversion, we want to outline the circulation of the energy in the vacuum: The propagation of an electric as well as a magnetic field influences the wavelength of the zero point oscillations. We will find the correlation between the field strength and the alteration of the wavelength soon. The central assumption of the model is that quantum electrodynamical corrections such as vacuum polarisation do not only occur with photons but also with zero point oscillations, and thus they extract energy from the field. But the particles of vacuum polarisation do not follow the propagation of the field, and so they can distribute their energy all over the space. This is the drain into which the field loses its energy during its propagation. On the other hand, this mechanism also gives us the explanation of the source, from

which the electrical charges are permanently supplied with energy in order to produce field strength.

This means that the reason for the transportation of field energy is an alteration of the wavelength of the zero point oscillations. The loss of energy which the field undergoes during its propagation as well as the mechanism of transporting the energy to the field source has the reason of vacuum polarisation. In the following chapter 3 and 4 we will see, that this model does not only explain the experiments of conversion of vacuum energy but it furthermore also allows us the determination of the energy density of the zero point oscillations in the vacuum.

3. New microscopic model for of the electromagnetic part of the vacuum-energy

The search for the connection between electric magnetic field energy on the one hand and vacuum-energy on the other hand, begins with a search of the relevant items inside the vacuum. There are the zero point oscillations mentioned above and besides there are several effects of vacuum polarisation (creating for instance virtual electrons and positrons) [17,18]. The logical consequence is clear: The energy of the vacuum as well as our mechanism of energy conversion must be explained on the basis of such items. With other words: We are searching for an explanation of the propagation of electric and magnetic fields, as well as for the supply of the field sources with energy, on the basis of these items.

A possible model therefore is surprisingly clear and simple, as following: The first thoughts go back to the year 1935, when Heisenberg and Euler [19] performed quantum theoretical calculations of the Lagrangian of photons in electric and magnetic fields, coming to the conclusion that photons propagate in such fields with lower speed than in vacuum without field. The reasons are found in vacuum polarisation, which influences the Lagrangian as calculated by Heisenberg and Euler. The experimental verification is not yet completely done. It was regarded as complete in [20], but this scientist later withdrew his results [21] by himself. But it is supposed, that the verification will be done in not too far future [22,23,24].

Logical consequence leads us to a conclusion, which is the only assumption of our model: If electromagnetic waves (as the photon) undergo retarded propagation within electric and magnetic fields, zero point oscillations should undergo the same retardation of propagation, because they have the same nature (to be electromagnetic waves). This means that electric and magnetic fields have an influence on the wave vector \mathbf{k} and on the frequency ω of the zero point oscillations. This causes an influence on the energy eigen-values of the zero point oscillations.

Consequently the alteration of the energy of the zero point oscillations should be identical with the energy of electric respectively the magnetic fields causing these alteration. This working hypothesis will be checked and confirmed by quantitative calculations on the following pages. The calculations are inspired by Casimir's considerations regarding the well known effect with his name. In analogy to his calculations we have to compare the spectrum of the zero point oscillations with and without an intervention. Casimir's intervention is to mount two conductive plates. In our case the intervention is to mount two conductive plates and additionally to apply an electric or a magnetic field. And now we have to compare the total energy of the spectra of the zero point oscillations in the space with and without field. The difference of those both total energy sums should be directly the energy of the field. In principle our calculation has the same problems with the convergence of improper integrals as Casimir's calculation. And the similar problems should be solvable in a similar way. The typical method for the solution known from Quantum field theory [25,26], see also [27,28,29]. The mathematical methods are presented very clearly in [30]. But the result can be obtained much easier, if we can find and use utilisable results somewhere in literature as done in the following calculation:

The energy density of the electromagnetic field of the zero point oscillations is calculated oftentimes in the \mathbf{k} -space [25], making use of $\mathbf{p}=(h/2\pi)\cdot\mathbf{k}$ as leading to the commonly known equation

$$(04) \quad \frac{E}{V} \Big|_Z = s \cdot \int E_0(\mathbf{k}) \frac{d^3k}{(2\pi)^3},$$

where $E_0(\mathbf{k})$ is the spectrum of the energy of the zero point oscillations, so that the integration is going continuously over all possible \mathbf{k} -vectors. The index "Z" stands for "zero point oscillations". The vacuum-energy is related to the transition-amplitude $\langle 0|0\rangle$ from the vacuum to the vacuum, which is represented by close loops for virtual particles in Feynman diagrams. The factor "s" is 1 because we want to consider the different states of polarisation separately. They correspond to the energy eigen-values of $E_0(\mathbf{k})=(n+1/2)\cdot(h/2\pi)\omega$, where $n=0$ is the ground state. So we have to insert $E_0(\mathbf{k})=1/2\cdot(h/2\pi)\omega$ into (4) and we receive

$$(05) \quad \frac{E}{V} \Big|_Z = \int \frac{1}{2} \hbar \omega \frac{d^3k}{(2\pi)^3}.$$

Furthermore the isotropy of the space allows us to work with the absolute values of the \mathbf{k} -vector and to write $\omega=c\cdot|\mathbf{k}|$, or in

Cartesian coordinates $\omega=c \cdot (k_x^2+k_y^2+k_z^2)^{1/2}$. Thus we come from (5) to

$$(06) \quad \frac{E}{V}|_Z = \frac{1}{2} \cdot \int \hbar c \cdot \sqrt{k_x^2+k_y^2+k_z^2} \frac{dk_x \cdot dk_y \cdot dk_z}{(2\pi)^3} = \frac{1}{2} \hbar c \cdot \int |\vec{k}| \frac{d^3k}{(2\pi)^3}$$

The divergence of the improper integral is commonly known, because the wave vector \mathbf{k} goes to infinity for small wavelength – and all these wave vectors have to be taken into account.

In analogy to Casimir's thoughts, we are not mainly interested (as explained above) in the limes of this integral, but we are mainly interested in the difference of the limes of this integral for \mathbf{k} -vectors with and without field. This means, we want to know the limes of

$$(07) \quad \frac{E}{V}|_{FIELD} = \frac{E}{V}|_{Z,WITH} - \frac{E}{V}|_{Z,WITHOUT} = \left(\frac{1}{2} \hbar c \cdot \int |\vec{k}_{Z,WITH}| \frac{d^3k}{(2\pi)^3} \right) - \left(\frac{1}{2} \hbar c \cdot \int |\vec{k}_{Z,WITHOUT}| \frac{d^3k}{(2\pi)^3} \right)$$

where the indices "WITH" and "WITHOUT" represent the situation with and without external field. This difference must be the energy density of the field, as marked by the index "FIELD". Equation (7) is our main interest for the explanation of the rotor converting vacuum-energy, but additionally it would be nice if we can find a way to solve equation (5) – and we will find it.

And this is the criterion of evaluation of our model: The model has to allow the calculation of the energy density of the zero point oscillations of the vacuum $E/V|_Z$ for both types of fields (electric as well as magnetic), and the energy density of the vacuum must be the same for both types of fields. The field can be regarded like a probe to stimulate the zero point oscillations, but the vacuum itself is the same, no matter which type of field is applied. The model can only be sensible, if both ways of calculation confirm each other – and this is, what we will find.

We prepare this evaluation with an introductory remark, applicable for both ways of calculation, before we come to each way in a separate consideration. [31] gives the Heisenberg-Euler-Lagrangian from [19], but he expresses it in SI-units (for the sake of understandability):

$$(08) \quad \mathcal{L} = -\frac{c^2 \epsilon_0}{4} F_{\mu\nu} F^{\mu\nu} + \frac{\alpha^2 \hbar^3 \epsilon_0^2}{90 m_e^4 c} \left[(F_{\mu\nu} F^{\mu\nu})^2 + \frac{7}{4} (F_{\mu\nu} \tilde{F}^{\mu\nu})^2 \right] = \frac{\epsilon_0}{2} (\vec{E}^2 - c^2 \vec{B}^2) + \frac{2\alpha^2 \hbar^3 \epsilon_0^2}{45 m_e^4 c^5} \left[(\vec{E}^2 - c^2 \vec{B}^2)^2 + 7c^2 (\vec{E} \cdot \vec{B})^2 \right]$$

where m_e is the mass of the electron and the other symbols are interpreted as usually.

There are several articles in which the speed of propagation of electromagnetic waves in electric, magnetic and electromagnetic DC-fields are calculated on the basis of this Heisenberg-Euler-Lagrangian. And we want to assign this speed of propagation also to the electromagnetic waves of the zero point oscillations of the vacuum. We will take the speed of propagation from those articles (which can already be found in literature) to determine the influences of the external field (electric and magnetic) onto the \mathbf{k} -vector of the zero point oscillations and thereby the influence on energy density of the zero point oscillations of the vacuum $E/V|_Z$. These results contain the solution of the improper integrals mentioned above. This shall be done now for both types of fields separately:

First calculation → Determination of $E/V|_Z$ with a magnetic field as a probe:

[31] gives the influence of a magnetic field on the speed of propagation of electromagnetic waves as

$$(09) \quad 1 - \frac{v}{c} = a \cdot \frac{\alpha^2 \hbar^3 \epsilon_0}{45 m_e^4 c^3} \cdot |\vec{B}|^2 \cdot \sin^2(\theta) = \begin{cases} 5.30 \cdot 10^{-24} \frac{1}{T^2} \cdot |\vec{B}|^2 \cdot \sin^2(\theta) & \text{for } a=8, \parallel\text{-mode} \\ 9.27 \cdot 10^{-24} \frac{1}{T^2} \cdot |\vec{B}|^2 \cdot \sin^2(\theta) & \text{for } a=14, \perp\text{-mode} \end{cases} \quad \left(\text{with } |\vec{B}| \text{ in Tesla} \right)$$

where the direction of the propagation of the photon and the direction of the magnetic field enclose an angle of θ , and they define a plane, which is the reference to classify the parallel-mode ($a=8$) and the perpendicular-mode ($a=14$). We have two different speeds of propagation, namely "v" with external field and "c" without external field. The difference of both speeds

leads us to the Cotton-Mouton-birefringence of the vacuum, being

$$(10) \quad \Delta n_{\text{Cotton-Mouton}} = \left(1 - \frac{\nu}{c}\right)_{\perp} - \left(1 - \frac{\nu}{c}\right)_{\parallel} = 3.97 \cdot 10^{-24} \frac{1}{T^2} \cdot |\vec{B}|^2 \cdot \sin^2(\theta)$$

which is confirmed by [32] quantitatively for an angle of $\theta=90^\circ$ and also by [33]. The last-mentioned reference is often regarded as a milestone on the way to the comprehension of electromagnetic waves in electric and magnetic DC-fields, because it is the first work giving quantitative predictions of the birefringence (and thus of the speed of propagation) of electromagnetic waves in the fields. This is a quantity which can be really measured.

The connection between the frequency and the speed of propagation of electromagnetic waves can be concluded from the fact, that the range of the integration over the \mathbf{k} -vectors is the same with and without the field, and thus $\mathbf{k}=\omega/c$ is independent of the question whether a field is applied or not. So we have the relation of the ratio

$$(11) \quad \frac{\omega_{\text{WITHOUT}}}{c} = \frac{\omega_{\text{WITH}}}{\nu} \quad \Rightarrow \quad \omega_{\text{WITH}} \cdot c = \omega_{\text{WITHOUT}} \cdot \nu \quad \Rightarrow \quad \omega_{\text{WITH}} = \omega_{\text{WITHOUT}} \cdot \frac{\nu}{c}$$

with "c" as the speed of propagation without field and "v" as the speed of propagation with field. The energy density of the magnetic field is known from classical electrodynamics [34] to be

$$(12) \quad \frac{E}{V} \Big|_{\text{FIELD}} = \frac{\mu_0}{2} \cdot \vec{H}^2 = \frac{1}{2\mu_0} \cdot \vec{B}^2$$

Equations (5) und (7) together with (11) lead us to the expression

$$(13) \quad \begin{aligned} \frac{1}{2\mu_0} \cdot |\vec{B}|^2 &= \frac{E}{V} \Big|_{\text{FIELD}} = \int \frac{1}{2} \hbar \cdot \omega_{Z, \text{WITHOUT}} \frac{d^3k}{(2\pi)^3} - \int \frac{1}{2} \hbar \cdot \omega_{Z, \text{WITH}} \frac{d^3k}{(2\pi)^3} \\ &= \int \frac{1}{2} \hbar \cdot \omega_{Z, \text{WITHOUT}} \frac{d^3k}{(2\pi)^3} - \int \frac{1}{2} \hbar \cdot \omega_{W, \text{WITHOUT}} \cdot \frac{\nu}{c} \frac{d^3k}{(2\pi)^3} \\ &\Rightarrow \frac{1}{2\mu_0} \cdot |\vec{B}|^2 = \frac{E}{V} \Big|_{\text{FIELD}} = \left(1 - \frac{\nu}{c}\right) \cdot \left(\int \frac{1}{2} \hbar \cdot \omega_{Z, \text{WITHOUT}} \frac{d^3k}{(2\pi)^3} \right) \end{aligned}$$

because the external fields alter the frequency and the energy of each quantum mechanical zero point oscillation, according to our model.

With (9) we find the sum of the energy of all zero point oscillations, because the field strength of the magnetic field is dispensed from the equation:

$$(14) \quad \begin{aligned} \frac{1}{2\mu_0} \cdot |\vec{B}|^2 &= \left(1 - \frac{\nu}{c}\right) \cdot \int \frac{1}{2} \hbar \cdot \omega_{Z, \text{WITHOUT}} \frac{d^3k}{(2\pi)^3} = a \cdot \frac{\alpha^2 \hbar^3 \epsilon_0}{45 m_e^4 c^3} \cdot |\vec{B}|^2 \cdot \sin^2(\theta) \cdot \int \frac{1}{2} \hbar \cdot \omega_{Z, \text{WITHOUT}} \frac{d^3k}{(2\pi)^3} \\ &\Rightarrow \int \frac{1}{2} \hbar \cdot \omega_{Z, \text{WITHOUT}} \frac{d^3k}{(2\pi)^3} = \frac{1}{2\mu_0} \cdot \frac{1}{a} \cdot \frac{45 m_e^4 c^3}{\alpha^2 \hbar^3 \epsilon_0} = \frac{1}{a} \cdot \frac{45 m_e^4 c^5}{2 \cdot \alpha^2 \hbar^3} = \frac{1}{a} \cdot 6.007 \cdot 10^{30} \frac{\text{J}}{\text{m}^3} \end{aligned}$$

at an excitation with $\theta=90^\circ$, where m_e is the mass of the electron and "alpha" is the constant of hyperfine structure.

Free from any problems of convergence of improper integrals, and free from any cut-off functions, we found in literature, this should be the energy density of the zero point oscillations of the vacuum. The problems of convergence have been solved in the work, from which we extracted (9). We have in mind that the calculation was done with a magnetic field as a probe, but the influence of the magnetic field was eliminated during the course of the calculation.

We also see, that the perpendicular-modes and the parallel-modes can be excited with different strength, but this is also an aspect of the magnetic field as a probe. It must not influence the answer to the fundamental question about the energy density of the zero point oscillations of the vacuum. Thus we have to extract a measurable quantity from our result, which will allow a comparison with the result of the second way of calculation. Such a quantity is the birefringence of the vacuum, which several articles regard as the central quantity of measurement.

We achieve the comparability as following: Also the difference, which represents the measurable birefringence must lead to the same energy density of the vacuum, this is

$$\begin{aligned}
 & \alpha_{\perp} \cdot \left[\int \frac{1}{2} \hbar \cdot \omega_{Z,WITHOUT} \frac{d^3k}{(2\pi)^3} \right] - \alpha_{\parallel} \cdot \left[\int \frac{1}{2} \hbar \cdot \omega_{Z,WITHOUT} \frac{d^3k}{(2\pi)^3} \right] = \frac{45m_e^4 c^3}{\alpha^2 \hbar^3 \epsilon_0} \\
 \Rightarrow & (14-8) \cdot \left[\int \frac{1}{2} \hbar \cdot \omega_{Z,WITHOUT} \frac{d^3k}{(2\pi)^3} \right]_{\perp-\parallel} = \frac{45m_e^4 c^5}{2 \cdot \alpha^2 \hbar^3} = 6.007 \cdot 10^{29} \frac{J}{m^3} \\
 (15) \Rightarrow & \left[\int \frac{1}{2} \hbar \cdot \omega_{Z,WITHOUT} \frac{d^3k}{(2\pi)^3} \right]_{\perp-\parallel} = \frac{1}{\alpha_{\perp} - \alpha_{\parallel}} \cdot \frac{45m_e^4 c^5}{2 \cdot \alpha^2 \hbar^3} = \frac{1}{14-8} \cdot 6.007 \cdot 10^{29} \frac{J}{m^3} = 1.001 \cdot 10^{29} \frac{J}{m^3}
 \end{aligned}$$

We got this value from the birefringence of electromagnetic waves in magnetic fields. We want to keep it in mind for later comparison with the birefringence of electromagnetic waves in electric fields, which will be the result of the second way of calculation.

To avoid confusion: The energy density which we calculated here is the energy density of electromagnetic zero point oscillations of the vacuum. This is not the total energy density of the vacuum, because it does not take the energy of other fundamental interactions into account (such as gravitation, weak and strong interaction). They are not matter of this article.

Second calculation → Determination of E/V_z with an electric field as a probe:

According to [32] the Kerr-birefringence of electromagnetic waves in electric fields has the absolute value of

$$(16) \quad \Delta n_{Kerr} \approx 4.2 \cdot 10^{-41} \frac{m^2}{V^2} \cdot |\vec{E}^*|^2$$

(with the electric field strength in V/m), with a possible uncertainty due to truncation of the numerical value. The value is also confirmed by [33].

From there we can easily find the absolute value of the energy density in analogy with (15) as following

$$\begin{aligned}
 & \frac{\epsilon_0}{2} |\vec{E}^*|^2 = \Delta n_{Kerr} \cdot \left[\int \frac{1}{2} \hbar \cdot \omega_{Z,WITHOUT} \frac{d^3k}{(2\pi)^3} \right]_{\perp-\parallel} \\
 (17) \Rightarrow & \left[\int \frac{1}{2} \hbar \cdot \omega_{Z,WITHOUT} \frac{d^3k}{(2\pi)^3} \right]_{\perp-\parallel} \approx \frac{\frac{\epsilon_0}{2} |\vec{E}^*|^2}{-4.2 \cdot 10^{-41} \frac{m^2}{V^2} \cdot |\vec{E}^*|^2} \approx 1.0 \cdot 10^{29} \frac{J}{m^3}
 \end{aligned}$$

This is the energy density difference for birefringence, determined with an electric field as a probe. Of course the field strength as an attribute of the probe had to be eliminated, so that the result is free from information about the probe. Thus (17) should come to the same result as (15), because there is only one vacuum, for which both calculations are valid. The fact, that this criterion is fulfilled demonstrates, that we have indeed found a possibility to trace back the convergence problems of (4), (5), (6) to other results found in literature. And it confirms our model of the propagation of electric and magnetic DC-fields with the model assumptions as described above.

For the sake of completeness, it should be mentioned again, that we calculated only the energy density of the electromagnetic zero point oscillations of the vacuum. This is not a general value for the total energy density of the vacuum. This is even not a value on the basis of higher order corrections of events of vacuum polarisation. And it does not contain any

considerations of fundamental interactions other than the electromagnetic interaction. We should not forget, that we try to explain the conversion only of the electric and the magnetic part of the vacuum energy in this article.

4. Dissolving the paradoxes of the electric and the magnetic field by the methods of a QED-model of the zero point oscillations. Functioning principle of the rotor for the conversion of vacuum energy.

Up to now, the model which was introduced in chapter 3 is capable to explain the propagation of electric and the magnetic fields in the vacuum. The only assumption it needs is surprisingly simple and plausible: From several articles in literature it is known, that the propagation of excited states $|n\rangle$ of electromagnetic waves (for $n \geq 1$) in the vacuum is influenced by electric and magnetic fields. The assumption of our model is, to apply the same dependence on electric and magnetic fields also to the propagation of the ground state (for $n=0$). This assumption looks logic and plausible. One of the consequences is, that the propagation of electric and magnetic DC-fields can be understood as an alteration of the wavelength of the ground state (these are the zero point oscillations).

But there is a further aim of the article: We want to explain the above mentioned energy-flux of the electric and the magnetic field and their propagation. And we want to explain the functioning principle of the rotor for the conversion of vacuum energy into classical (mechanical) energy. This can be done as following:

Let us begin with the first aspect, the energy-flux of the electric and the magnetic field. This is the question in which way the space absorbs energy from the propagating field and in which way it supplies the field source (this is the charge) with energy:

The quantum electrodynamic corrections in the Heisenberg-Euler-Lagrangian, as used in chapter 3, correspond to terms of energy (otherwise they would not be part of the Lagrangian). Each energy correction corresponds to an event of vacuum polarisation. The corrections which Heisenberg and Euler took into account do this in the same manner as further corrections of higher order corresponding to different effects of vacuum polarisation. Also the zero point oscillations have their contribution to these terms of energy (otherwise they would not be influenced by the fields). Although events of vacuum polarisation only occur temporarily, there are always some of these events going on, depending on their probability amplitudes. The situation describes a permanent (statistical) coming and going reminding to a picture of people coming and going along a street. Although they stay only temporarily in the street, you can always see people there. As long as events of vacuum polarisation occur, there exist some virtual electrons and positron and so on...

With other words: In our model, the field generates a lot of events of vacuum polarisation during its propagation (because of the field's energy) – and this is just the amount of energy which the field gives to the vacuum during propagation, so that the field strength will follow Coulomb' law. The events of vacuum polarisation are the medium which extracts energy from the field.

The way how the field source is supplied with energy follows the same principle but to the opposite direction: The events of vacuum polarisation take finite time and finite space, but they don't have to follow the directions of the flux lines of field. They diffuse to every direction and to everywhere in the space. Obviously they generate the loss of energy from the field during propagation (see (2) and (3)). Some of them come close to an electrical charge (to a field source), and the charge extracts energy from them. It is not the issue here, to think about the mechanism of this extraction. It is even not said, that the fields source gets back the energy from its own field. But it is clear that the field source is supplied with energy, which does not originate from its own field, because the total energy of the field increases during time as long as the field propagates into the space. This is clear, because the volume filled with field is permanently increasing during time.

In the notation of particle physics, the zero point oscillations are bosonic Quantum field fluctuations (because they are electromagnetic waves), and events of vacuum polarisation are fermionic Quantum field fluctuations (because they consist of particles like virtual electrons and positrons). Their conversion (which is necessary for vacuum polarisation) consists of processes like virtual pair production and annihilation. Especially in electric and magnetic fields the probability amplitudes for the conversion for those both types of Quantum field fluctuations depend on external field strength. Thus the distance to the field source has an influence on the number of such processes occurring in a given time interval.

Under these circumstances we can calculate the energy loss of a field produced by a point charge during its propagation (which should be connected with the probability amplitudes of the events of vacuum polarisation). This can be done even with a simple classical calculation on the basis of (2) and (3) and fig.1:

When a spherical shell from $x_1 \dots x_1 + c \cdot \Delta t$ filled with field propagates to $x_2 \dots x_2 + c \cdot \Delta t$, it releases the energy

$$(18) \quad \Delta E = E_{inner} - E_{outer} = \frac{Q^2}{8\pi\epsilon_0} \cdot \frac{c \cdot \Delta t}{(x_1 + c \cdot \Delta t) \cdot x_1} - \frac{Q^2}{8\pi\epsilon_0} \cdot \frac{c \cdot \Delta t}{(x_1 + \Delta x + c \cdot \Delta t) \cdot (x_1 + \Delta x)}$$

For small but finite Δt and Δx (so that they can be neglected as summands in comparison with values devoid of Δ ..., but not as factors), we come to the approximation.

$$(19) \quad \Delta E = E_{\text{inner shell}} - E_{\text{outer shell}} = \frac{Q^2}{8\pi\epsilon_0} \cdot \frac{(2x_1 + \Delta x + c \cdot \Delta t) \cdot c \cdot \Delta t \cdot \Delta x}{(x_1 + c \cdot \Delta t) \cdot x_1 \cdot (x_1 + \Delta x + c \cdot \Delta t) \cdot (x_1 + \Delta x)} \approx \frac{Q^2}{8\pi\epsilon_0} \cdot \frac{2x_1 \cdot c \cdot \Delta t \cdot \Delta x}{x_1 \cdot x_1 \cdot x_1 \cdot x_1} = \frac{Q^2}{8\pi\epsilon_0} \cdot \frac{2 \cdot c \cdot \Delta t \cdot \Delta x}{x_1^3}$$

To determine the energy transfer, the thickness of the shell $c \cdot \Delta t$ has to be finite (otherwise there would no be a shell at all) as well as the distance of propagation Δx (because for $\Delta x \rightarrow 0$ there would not be propagation at all), and then we find that the shell disperses energy with reciprocal proportionality to the third (!) power of the radius x_1 of the shell. Now we can subsume the systematics of the proportionalities of some field-parameters with regard to a point charge and its field (with "r" being the distance from the point charge) as following:

Field-parameter	Proportionality
Electric Potential of a point charge	$V \sim r^{-1}$
Field strength caused by a point charge (Coulomb's law)	$F \sim r^{-2}$
Dispersed energy of a spherical shell around a point charge	$E \sim r^{-3}$
Energy density of the field of a point charge	$u \sim r^{-4}$

Comment: The number of events of vacuum polarisation per space and time (ultimately their probability amplitudes) is responsible for the amount of energy, which the field gives to the space.

Let us come to the understanding of the functioning principle of the rotor, which is already experimentally proven to convert vacuum-energy into mechanical: We want to explain the mechanism according to which this rotor works:

Of course the answer shall be based on the energy of the electrostatic field (or the magnetic field in the case of a magnetically driven rotor), because the forces have been calculated as electrostatic forces successfully. According to our model the rotor is mounted inside the energy flux going from the field source into the space. So obviously the rotor extracts energy from this energy flux – in detail as following:

The field source produces an electric field, thereby it emanates an energy flux which causes an alteration of the wavelengths of the zero point oscillations (see above). As we know from the Casimir-effect, conducting plates block the propagation of the zero point oscillations. If we adopt this principle to our experiment, this means that between the field source and the rotor, we have the alteration of the wavelengths. But on the other side of the rotor blades, looking away from the field source, the zero point oscillations have the wavelengths as in free space without field, because this part of the space, which is shielded by the rotor blades against the field source. Thus the wavelengths on the one side of the rotor blades differ from the wavelengths on the other side of the rotor blades. From the point of energy conservation, this is only possible, if the rotor blades compensate the difference of energy. In fact the rotor blades absorb energy (they do not emit energy), because the total energy of the field is reduced in comparison to the situation without rotor blades, because the field produced by the field source does not reach the space behind the rotor blades.

In consequence this means that the energy flux is going from the field source to the rotor blades, which absorb some energy from the flux. The field source converts vacuum energy into field energy (this is the energy to alter the wavelengths of the zero point oscillations), which is a flux going into the space. This part of the flux which meets the rotor blades is absorbed from the rotor blades, being converted into mechanical energy. By the way: If someone wants to calculate the forces onto the rotor blades from the alteration of the wavelengths, it would be necessary to consider the retroactive action from the rotor blades on the wavelengths. In this context we know the image- charge method in classical electrodynamics [35].

5. Perorating discussion

The model presented here was developed on the basis of commonly accepted fundamental elements of Quantum theory plus one very plausible assumption that the propagation of the zero point electromagnetic waves have the same behavior in electric and magnetic fields as other (non zero point) electromagnetic waves. Certainly the calculus of our model with the methods of Quantum electrodynamics and/or Quantum field theory should be further developed. This should be hopefully a good way, because the model is already experimentally confirmed.

Let us compare our results of the energy density of the vacuum (i.e. the space) with energy density values from other theories or measurements. But when we do this, we should not forget, that the energy density which we calculated within our model is only due electromagnetic zero point oscillations (as given in chapter 3). This is not the same as the energy density found in cosmology.

The question of the energy density of the universe is known to be one of the important unsolved puzzles in nowadays physics, and it is the largest discrepancy known in our days with a range of more than 120 orders of magnitude between

different values from different branches on physics. On the one hand we can find a many workings of cosmology (see for instance [36,37,38,39,40]), giving values of the density of matter ρ_m and/or the energy density ρ_{grav} of the universe based on considerations of gravitation in connection with the expansion of the universe. In the average most of them come to values in the range of

$$(20) \quad \rho_M \approx (1.0 \pm 0.3) \cdot 10^{-26} \frac{kg}{m^3} \Rightarrow \rho_{grav} = c^2 \cdot \rho_M = (9.0 \pm 0.27) \cdot 10^{-10} \frac{J}{m^3}$$

The fundamentals of the expansion of the universe are experimental observations. On the other hand there are values coming from Geometrodynamics [41], which are determined by pure theoretical considerations, based on the zero point oscillations of the vacuum plus the hypothesis that zero point oscillations with a wavelength below the Planck-scale can not exist. The basis of this hypothesis is just the assumption that it might be like this, there is no physical reason for it. The background of this hypothesis is simply the idea to get rid of the convergence problems of the improper integral in (5) by restricting the integral to a finite range. With the value of the Planck-length of $L_p = (\hbar \cdot G / c^3)^{1/2} \approx 4.05 \cdot 10^{-35} m$ [Tip 03], we can just alter (5) and calculate its value:

$$(21) \quad \left. \frac{E}{V} \right|_{GD} = 2 \cdot \int_{|\vec{k}|=0}^{\frac{2\pi}{L_p}} E_0(|\vec{k}|) \frac{d^3k}{(2\pi)^3} = 2 \cdot \int_{|\vec{k}|=0}^{\frac{2\pi}{L_p}} \frac{1}{2} \hbar \omega \frac{d^3k}{(2\pi)^3} = 2 \cdot \underbrace{\int_0^{\frac{2\pi}{L_p}} \frac{1}{2} \hbar c |\vec{k}| \cdot |\vec{k}|^2 \frac{d k}{2\pi^2}}_{\substack{\text{Umrechnung in Kugel-} \\ \text{koordinaten nach [KUH95]}} = 2 \cdot \frac{\hbar c}{4\pi^2} \cdot \frac{1}{4} \left(\frac{2\pi}{L_p} \right)^4 = \frac{2 \hbar c \pi^2}{L_p^4} = 3.32 \cdot 10^{+113} \frac{J}{m^3}$$

(The factor of 2 in front of the integral represents two states of polarisation.):

But how can we interpret the discrepancy between (20) and (21) ?

Well – the result of (21) is mostly regarded very sceptically, because there is not a serious argument for the way, how the convergence problems was suppressed. In principle the idea of our model also begins with (5), but we have found a serious physical argument to solve the convergence problems.

The interpretation of the values from cosmology and astrophysics in (20) is the following: If the value of our model is larger than the value of (20) – can our values be possible in such a case (which is given here) ? The problem is gravitation: If our value would be correct – shouldn't this mean, that the attractive forces of gravitation should be larger than the observed ones ?

This conclusion is not possible, thus it does not lead to a contradiction. Reason: The values of (20) are deduced from the expansion of the universe, where the universe is regarded as a spherical distribution of matter (and dark matter) with a radius taken from the hypothesis, that the beginning of the universe was the "big bang" and the expansion of the matter and the dark matter is for sure not faster than the speed of light [43]. However our values coming from the energy density of the zero point oscillations refer to the whole space R^3 , hence also the space outside the sphere which we call universe. For gravitation this difference is definitely important. And furthermore there is an other problem with the value from cosmology, namely the unsolved question of the accelerated expansion of the universe [44], which leads to uncertainties in the background of (20).

Thus we do not face a contradiction between our value (which is only a part of the energy density of the vacuum) and other values. We should rather interpret our value like this:

The calculation of the energy density of the zero point oscillations of the vacuum runs into convergence problem with an improper integral, leading to an infinite energy density. Classical Quantum theory goes around this problem by ignoring the infinity and fixing the energy scale on top this infinite energy. This perception is not fully satisfying, so Geometrodynamics tries to cut off the improper integral artificially by taking a finite range for the integration. It is clear that this approach avoids problems of divergence by principle, but it results in coming to rather strange values. A physical solution for this divergence problem is found in the work presented here, and it leads to values looking sensible.

References:

1. Mandl, F. and Shaw, G. (1993). Quantenfeldtheorie. Aula-Verlag, ISBN 3-89104-532-8; English version available: Quantum Field Theory. John Wiley und Sons, ISBN 0-471-94186-7
2. Kuhn, W. and Strnad, J. (1995). Quantenfeldtheorie. Vieweg Verlag, ISBN 3-528-07275-X
3. Köpp, G. and Krüger, F. (1997). Einführung in die Quantenelektrodynamik. B.G.Teubner Verlag, ISBN 3-519-03235-X

4. Casimir, H. B. G. (1948). On the attraction between two perfectly conducting plates. Proceedings of the Section of Sciences, Koninklijke Nederlandse Akademie van Wetenschappen, page 795 and Casimir, H. B. G. and Polder, D. (1948). Phys. Rev. 73, page 360
5. Mohideen, U. and Roy, A. (1998). Precision Measurement of the Casimir Force from 0.1 to 0.9 μm . Phys. Rev. Lett. Vol. 81, no.21, page 4549
6. Bressi, G., Carugno, G., Onofrio, R., Ruoso, G. (2002). Measurement of the Casimir Force Force between Parallel Metallic Surfaces. Phys. Rev. Lett. Vol. 88, no.4, S. 4549, S. 041804-1-4
7. Svetovoy, V. B. and Lokhanin, M. V. (2000). Precise Calculation of the Casimir Force between Gold Surfaces. Modern Physics Letters A, vol.15, nos. 22 & 23, S.1437
8. Turtur, C. W. (2. April 2008). Conversion of vacuum-energy into mechanical energy: Successful experimental Verification. PHILICA.COM, ISSN 1751-3030, Article number 124
9. Turtur, C. W. (5. Juni 2008). Review: Conversion of vacuum-energy into mechanical energy. The General Science Journal, ISSN 1916-5382
10. Turtur, C. W. (Oktober 2007). Two Paradoxes of the Existence of electric Charge. <http://arXiv.org/abs/0710.3253>
11. Turtur, C. W. (18. Februar 2008). A Motor driven by Electrostatic Forces. PHILICA.COM, ISSN 1751-3030, Article number 119
12. Klingbeil, H. (2003). Elektromagnetische Feldtheorie. Teubner Verlag, ISBN 3-519-00431-3
13. Goenner, H. (1997). Einführung in die spezielle und allgemeine Relativitätstheorie. Spektrum Akademischer Verlag, ISBN 3-86025-333-6
14. Chubykalo, A. E., Pope, V. and Smirnov-Rueda, R. (1999). Instantaneous Action at a Distance in Modern Physics: "Pro" and "Contra". Nova Science Publishers, ISBN1-56072-718-7
15. Landau, L. D. and Lifschitz, E. M. (1997). Lehrbuch der theoretischen Physik (especially vol. 2, Feldtheorie). Verlag Harri Deutsch, ISBN 3-8171-1327-7
16. Turtur, C. W. (19. December 2007). Two Paradoxes of the Existence of magnetic Fields. PHILICA.COM, ISSN 1751-3030, Article number 113
17. Feynman, R. P. (1997). Quantenelektrodynamik. Oldenbourg Verlag, ISBN 3-486-24337-3
18. Giacomo, A. Di, Dosch, H. G., Shevchenko, V. I., Simonov, Yu. A. (2000). Field correlators in QCD. Theory and Applications, arXiv:/hep-ph/0007223
19. Heisenberg, W. and Euler, H. (1936). Folgerungen aus der Diracschen Theorie des Positrons, Zeitschrift für Physik, page 714
20. Zavattini, E. et. al. (2006). Experimental Observation of Optical Rotation Generated in Vacuum by a Magnetic Field. Phys. Rev. Lett. 96, 110406
21. Zavattini, E. et. al. (Sept. 2007). New PVLAS results and limits on magnetically induced optical rotation and ellipticity in vacuum. arXiv:0706.3419v2
22. Sheng-Jui Chen, Hsien-Hao Mei, Wei-Tou Ni. (28. Nov. 2006). Q & A Experiment to search for vacuum dichroism, pseudoscalar-photon interaction and millicharged fermions. arXiv:hep-ex/0611050
23. Lamoreaux, S. (2006). The first axion ? Nature, Vol.441, pages 31-32
24. Beswick, J. A. and Rizzo, C. (2007). Structure of the photon and magnetic field induced birefringence and dichroism. arXiv:quant-ph/0702128v1
25. She-Sheng Xue. (2001). Possible vacuum-energy releasing. Physics Letters B 508
26. She-Sheng Xue. (2003). Magnetically induced vacuum decay. Phys. Rev. D 68 (2003) 013004
27. Hooft, G. 't and Veltman, M. (1972). Regularization and renormalization of gauge fields. Nuclear Physics B44, page 189
28. Dowker, J. S. and Banach, R. (1978). Quantum field theory of Clifford-Klein space-times. The effective Lagrangian and vacuum stress-energy tensor. J. Phys. A: Math. Gen., vol.11, no.11, page 2255
29. Blau, S. K., Visser, M. and Wipf, A. (1991). Analytic results for the effective action, International Journal of Modern Physics A, vol.6, no.30, S.5409
30. Kleinert, H. and She-Sheng Xue. (März 2008). Photoproduction in semiconductors by onset of magnetic field, EPL, 81 (2008) 57001
31. Boer, D. and van Holten, J.-W. (2007). Exploring the QED vacuum with laser interferometers. arXiv:/hep-ph/0204207v1
32. Rikken, G. L. J. A. and Rizzo, C. (2000). Magnetolectric birefringences of the quantum vacuum. Phys. Rev. A, Vol.63, 012107
33. Bialynicka-Birula, Z. and Bialynicki-Birula, I. (1970). Nonlinear Effects in Quantum Electrodynamics. Photon Propagation and Photon Splitting in an external Field. Phys. Rev. D, Vol.2, No.10, page 2341
34. Jackson, J. D. (1981). Klassische Elektrodynamik. Walter de Gruyter Verlag, ISBN 3-11-007415-X
35. Becker, R. and Sauter, F. (1973). Theorie der Elektrizität. Teubner Verlag, ISBN 3-519-23006-2
36. Giulini, D. and Straumann, N. (22. Sep. 2000). Das Rätsel der kosmologischen Vakuumenergiedichte und die beschleunigte Expansion des Universums. arXiv:astro-ph/0009368 v1
37. Tegmark, M. (10. Jul. 2002). Measuring Spacetime: from Big Bang to Black Holes. arXiv:astro-ph/0207199 v1 ; Slightly abbreviated version in: Science, 296, 1427-1433 (2002)
38. Efstathiou, G. (Feb. 2002). The Monthly Notices of the Royal Astronomical Society, Volume 330, No. 2, 21.
39. Tonry, J. L., et. al. (1. Mai 2003). Cosmological Results from High-z Supernovae, arXiv:astro-ph/0305008
40. Riess, A. G., et. al. (1998). Observational Evidence from Supernovae for an Accelerating Universe and a Cosmological Constant. arXiv:astro-ph/9805201
41. Wheeler, J. A. (1968). Einsteins Vision - Wie steht es heute mit Einsteins Vision, alles als Geometrie aufzufassen ? Springer Verlag, Berlin
42. Tipler, P. A. and Llewellyn, R. A. (2003). Moderne Physik. Oldenbourg Verlag, ISBN 3-486-25564-9
43. Perlmutter, S. et. al. (8. Dez. 1998). Measurements of and from 42 high-redshift Supernovae. arXiv:astro-ph/9812133,
44. Célérier, M.-N. (7. Jun. 2007). The Accelerated Expansion of the Universe Challenged by an Effect of the

